From using artefacts to mathematical meanings: the teacher's role in the semiotic mediation process

Dall'utilizzo degli artefatti ai significati matematici: il ruolo dell'insegnante nel processo di mediazione semiotica

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Abstract / The didactic potential of artefacts for learning have been extensively studied, with a main focus on their possible use by students and the subsequent benefits for them. However, there has been the tendency to underestimate the complexity of exploiting this potential, and specifically the complexity of the teacher's role orchestrating the teaching and learning process. Following Vygotskij's seminal idea of semiotic mediation, the theoretical framework of Theory of Semiotic Mediation (TSM) has been developed (Bartolini Bussi & Mariotti, 2008) with the aim of providing a teaching and learning model, where attention is focused on the semiotic processes related to the use of cultural artefacts. Through the semiotic lens it is possible to analyse the classroom discourse and highlight specific patterns in the teacher's action that make students' personal meanings evolve towards the mathematical meanings that are the objective of the didactic intervention. The paper presents a first model of the teacher's action and provides some examples drawn from long term teaching experiments carried out at the primary school level.

Keywords: semiotic mediation; artefact; semiotic potential; didactic cycle; teacher.

Sunto / Il potenziale didattico degli artefatti nell'apprendimento è stato ampiamente studiato, focalizzandosi soprattutto sul loro possibile uso da parte degli studenti e sui benefici che ne derivano. Vi è però la tendenza a sottostimare la complessità legata alla piena espressione di tale potenziale, e in particolare l'importanza del ruolo del docente nell'organizzazione del processo d'insegnamento e apprendimento. A partire dall'idea seminale di mediazione semiotica introdotta da Vygotskij, è stato sviluppato il quadro teorico della Teoria della Mediazione Semiotica (TMS) (Bartolini Bussi & Mariotti, 2008), con l'obiettivo di fornire un modello di insegnamento e apprendimento che pone l'attenzione sul processo semiotico relativo all'utilizzo di artefatti culturali. Assumendo una prospettiva semiotica è possibile analizzare il discorso che si sviluppa nella classe ed evidenziare specifici schemi di azione messi in atto dall'insegnante al fine di far evolvere i significati personali degli studenti verso i significati matematici che sono l'obiettivo dell'intervento didattico. L'articolo presenta un primo modello dell'azione del docente e fornisce alcuni esempi legati ad una sperimentazione didattica a lungo termine svolta a livello della scuola elementare.

Parole chiave: mediazione semiotica; artefatti; potenziale semiotico; ciclo didattico; docente.

1 Introduction

The main role of a mathematics teacher is to make mathematical knowledge accessible for her/his students. This educational task can be accomplished introducing various means in the mathematics classes. Manipulatives are a common example at

the primary level. ICT technologies can also serve for such purpose. However this tools can be useless without specific actions made by the teacher to mediate the mathematical content that is the aim of the designed teaching sequence.
Elaborating on the seminal idea of semiotic mediation introduced by Vygotsky (1978), Bartolini Bussi & Mariotti (2008) elaborated a model aimed at describing and explaining the process that starts with the student's use of a specific tool to accomplish a task and leads to the student's appropriation of a particular mathematical content. In this respect, such a model considers the issue of integrating tools from a broader perspective, and proposes a theoretical framework that might include any kind of artefact, given its potential to be related to a specific mathematical meaning; moreover, this theoretical approach explicitly takes into account the role of the teacher, offering the base for an explicit model of what is expected from her/him.
In this contribution, after giving an overview of the key ideas of the theoretical framework (the Theory of Semiotic Mediation, TSM) we will focus on the action of the teacher, on how she/he can utilize the artefact according to her/his specific educational goals.

2 Mediation According to a Semiotic Approach

The term *mediation* has been frequently used, with different and not always compatible meanings, in relation to the introduction of artefacts in school practice, and it has become widely present in the current mathematics education literature (Meira, 1995; Radford, 2003; Noss & Hoyles, 1996; Borba & Villarreal, 2005). Mostly used in relation to the support that a specific tool gives one in the accomplishment of a task through its use, the idea of mediation has been also related to the potentiality of fostering learning processes with respect to a specific piece of knowledge, for instance mathematical knowledge. However, often, the complexity of the mediation process has not been adequately addressed, as a consequence of neglecting the epistemological issue concerning the relationship between the accomplishment of a task and the student's mathematical learning processes.
On the contrary, this issue is specifically addressed by the Theory of Semiotic Mediation (TSM) (Bartolini Bussi & Mariotti, 2008), which elaborates on the notion of mediation combining a semiotic and an educational perspective, and considering the crucial role of *human mediation* (Kozulin, 2003, p.19) in the teaching-learning process. The TSM provide a model of the teaching and learning process developed around two key elements: the notion of semiotic potential of an artefact and the notion of didactic cycle. After a short description of these main notions, we will describe different mediation modalities that may be accomplished by the teacher in exploiting the semiotic potential of a given artefact within the collective phase of a didactic cycle.

3 The semiotic potential of an artefact

The relationship between mathematical knowledge and the use of specific tools has a long standing history: the case of ruler and compass is perhaps the most representative. As a matter of fact, observing the use of a specific tool it may happen that an expert is led to recall a specific mathematical knowledge. Following Hoyles (1993), one can speak about the relationship between artefact and knowledge as *evoked knowledge*. For example, positional notation and the polynomial notation of numbers may be evoked by an abacus and by its use; similarly, for a mathematician, a Dynamic Geometry System may evoke the classic "ruler and compass" Geometry. Let us consider the following tools (Figure 1): one is the well-known pair of compasses and the other is the combination of a glass and a pencil.

Figure 1
The pair of compasses and the glass & pencil.

When used to draw, both these tools produce a circular trace, a round, and for this reason, both of them can be related, and thus evoke, the mathematical notion of circle. However, in spite of the common final product, if we compare them with respect to the gestures acted, we observe that the followed procedure is completely different and it evokes quite different geometrical elements and properties.
According to Rabardel (1995), we distinguish between the object itself – the artefact – and the procedure that is used to accomplish the task – scheme of utilization. Considering the combination of an artefact and its scheme of utilization we can identify specific mathematical meanings that may be evoked by the use of a specific artefact. As for the combination of glass and pencil, the schema of utilization focuses on the circular movement of the hand holding the pencil; this leads to the regularity of the specific shape that for an expert may evoke its geometric property of constant curvature; in the case of the pair of compasses, the schema of utilization focuses on the presence of the special point where the tip of the first arm is fixed, the enlargement of the two arms and the constancy of such enlargement in tracing the round shape. For an expert, all that may evoke the notion of *centre*, the notion of *radius*, together with the property of the constancy of the distance between the centre and any point of the circle, that is the geometrical *definition of circle* as locus of points.
It is clear that depending on which of the two artefact is used, different mathematical meanings are evoked (Chassapis, 1998); in other words we can say that the two artefact have different *semiotic potential*. In general, we define the *semiotic potential of an artefact* as the double relationship that may occur between an artefact

and, on the one hand, the personal meanings emerging from its use to accomplish a task (instrumented activity), and on the other hand the mathematical meanings evoked by its use and recognizable as Mathematics by an expert (Bartolini Bussi & Mariotti, 2008).
An a priori analysis, involving at the same time a cognitive and epistemological perspective, may lead one to identify the *semiotic potential* of an artefact.
The design of any pedagogical plan (teaching-learning sequence) centred on the use of a given artefact has to be based on an a priori description of the semiotic potential of an artefact; examples inspired by classic ancient artefact can be that of the abacus (Bartolini Bussi & Mariotti, 2008, p. 758 ff.) and that of the prospectograph (Bartolini Bussi, Mariotti & Ferri, 2005).

4 The didactic cycle

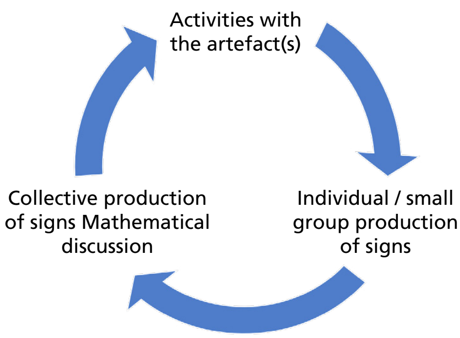

Figure 2
The didactic cycle.

The identification of the semiotic potential of a certain artefact constitutes the necessary ante fact of its use in the classroom, however the effectiveness of its use needs the careful design of the teaching intervention. When a cognitive and epistemological analysis has produced a sequence of possible tasks, a suitable didactic organization of these tasks is required, together with teacher's appropriate management of classroom activities. Taking a semiotic perspective means to focus on the production of signs and on the process of transformation of such signs, considering such transformation as an evidence of learning. In other words, according to the TSM we recognize a central role to signs[1], both as a product and as a medium, and we interpret the construction of knowledge as an evolution from meanings rooted in the use of the artefact toward meanings explicitly recognized as consistent with Mathematical meanings: starting with the *unfolding of the semiotic potential*, witnessed by students production of specific signs, the meaning of which primarily refers to the use of the artefact *(artefact signs)*, the active intervention of the teacher will promote the evolution of such signs into the expected mathematical signs. The whole structure of a teaching sequence may be outlined as an iteration of *didacti-*

1. The use of the term sign is inspired by Pierce. We assume an indissoluble relationship between signified and signifier. In the stream of other researchers (Radford, 2003; Arzarello, 2006) we developed the idea of meaning originated in the intricate interplay of signs (Bartolini Bussi & Mariotti, 2008); for a thoughtful discussion see also (Sfard, 2000, p. 42 and following).

cal cycles, designed to *exploit the semiotic potential of the artefact*. Each cycle is constituted by specific activities, each type of activity contributes differently, but complementarily, to the developing of the complex process of semiotic mediation.
In summary, *the semiotic mediation process* consists in the evolution from the emergence of personal meanings related to the accomplishment of a task to the collective development/construction of shared signs related to both the artefact's use and the mathematics to be learnt. A sequence of didactic cycles can be organized with the aim of supporting such evolution; the different activities proposed in each cycle are classified according to their contribution to the semiotic mediation process. Three categories of tasks constitute one didactic cycle, as described in the following.

Activities with the artefact. These constitute the beginning of any cycle and they are based on asking students to carry out a task involving the use of the artefact, with the aim of promoting the emergence of signs (words, sketches, gestures) whose meanings refers to the use of the artefact but are also coherent with the mathematical meanings that are the goal of the teaching intervention.

Activities of individual production of signs. Spontaneous production of signs emerges during the activities with the artefact, so different semiotic activities are proposed, asking individual production and elaboration of signs, related to the previous phase. A crucial role is played by written texts, because of their nature and unlike other signs, like gestures, written signs (in particular words) start to be detached from the contingency of the situated action. Moreover written productions can become objects of discussion in the following collective work.

Collective discussion. This kind of activity plays an essential part in the teaching-learning process and constitutes the core of the semiotic mediation process. The whole class is engaged: various solutions are discussed collectively, students' written texts or other texts are collectively analysed, commented, elaborated. Students' intervention are coordinated by the teacher with the objective of fostering the move towards mathematical meanings, exploiting the semiotic potentialities coming from the use of the particular artefact.

5 The role of the teacher in the development of the semiotic mediation process

The semiotic mediation perspective stresses the fact that the artefact is used in the classroom not only by the students for solving given tasks, but it is also used by the teacher for accomplishing a didactical task, pursuing her/his educational goals. In this sense, the artefact is a resource for the teacher. It is used by the teacher as a tool of semiotic mediation (Bartolini Bussi & Mariotti, 2008, p. 754).
In each phases of the didactic cycle the teacher plays a crucial role, her/his intervention concerns

- the design of tasks, aiming at favouring the unfolding of the semiotic potential of the selected artefact;
- the analysis of students' written solutions and reports after the accomplishment

of the tasks identifying the emergence of the expected signs;
– the planning of collective discussions on the base of the results of the previous analysis;
– the management of the collective discussion fostering the evolution of students' personal meanings towards the desired mathematical signs.

Some of the teacher's actions sketched above concern the design of the teaching interventions, while others concern its enactment; some concern the social activities in the classroom and others concern the individual activities.

In the following sections, we will focus on the teacher's actions concerning collective activities, specifically those aimed at driving the classroom discussion in order to promote the development of the semiotic mediation process.

5.1 The evolution of the mathematical discourse in classroom

Consistently with a Vygotskian approach, we interpret the teaching and learning process as a *process of internalization*; we interpret individual knowledge construction, as a social endeavour, directed by semiotic processes related to communication, and involving the production and interpretation of signs, in what can be called *interpersonal space* (Cummins, 1996). Moreover, still consistently with a Vygotskian approach, at the core of the TSM there is also the assumption that semiotic processes, generating and fostering the social construction of knowledge, may be directed by the teacher who can intentionally act to promote the evolution of the classroom mathematical discourse.[2] 2

A number of teaching experiments were carried out in the past years, involving different artefacts, of different nature; according to a design-based approach, where «the design is conceived not just to meet local needs, but to advance a theoretical agenda» (Barab & Squire, 2004, p. 5), specific attention was dedicated to analyse teacher's action in managing the process of evolution of signs (Mariotti, 2001; Cerulli, 2004; Falcade, Laborde & Mariotti, 2004; Falcade, 2006; Mariotti & Maracci 2010). Such analysis highlighted recurrent action patterns related to an effective development of semiotic mediation process; as a matter of fact, these patterns can be described according to the main objectives of the classroom discussion within a didactical cycle: that is, promoting both the emergence and sharing of the students' personal signs and their evolution towards the desired mathematical signs. The description of the different categories constituting the action patterns will be given in terms of these objectives.

In the following we present a description of the teacher's actions emerging from observations in the classroom and characterized according to a specific objective related to the expected evolution of signs; each type of action is illustrated by examples taken from a particular teaching experiment. In order to make the examples clearer, we start with a short overview of the teaching experiment.

An overview of the teaching experiment

Proposed examples are taken from a teaching experiment realized at the primary school level. The aim of the teaching experiment was the introduction of multiplica-

2. The use of the term *Mathematical discourse* is consistent with the Moschkovich's (2003). Characterization: «Mathematical Discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view. Participating in mathematical discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act when talking about mathematics».

tion properties according to a relational approach (Maffia, 2017), and the artefact on which was based the intervention consists in Laisant table (also known as *decanomial*). It is a table with rows and columns formed by cells of different size. Each cell of a row is one unit higher than those of the previous row. In the same way, each cell of a column is larger than the one on its left, as shown in **Figure 3**. During the long-term intervention children had the opportunity to explore the table, discuss the way in which it is made. They realized their personal table and used the table to represent multiplications and determine products. They were able to associate each cell with the corresponding multiplication, recognizing both the factors and the result. The table was also used to introduce the relation of equivalence of multiplications through the equivalence of rectangles that was checked by superposition of rectangular slip of paper realized by the children.

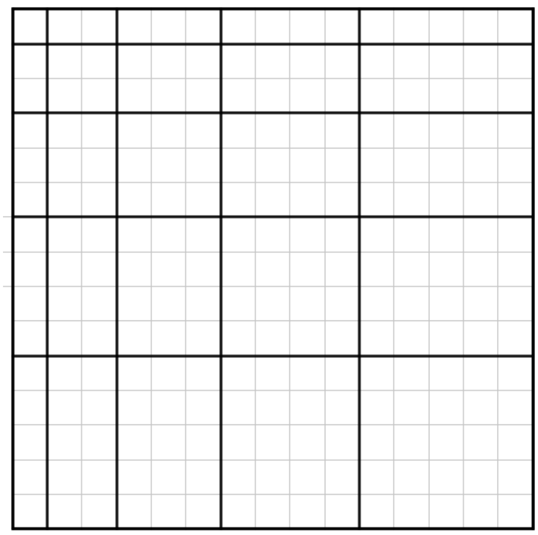

Figure 3
Laisant table with five rows and five columns.

Considering some tasks that can be faced by using this artefact, we can analyze its semiotic potential. It is possible to produce rectangular slip of paper, cut them, manipulate them and paste them again. In particular: rectangles can be decomposed and re-composed obtaining rectangles with different sides. Such operations can show that the surface of the rectangle (that is the result of the multiplication) does not change (**Figure 4**).

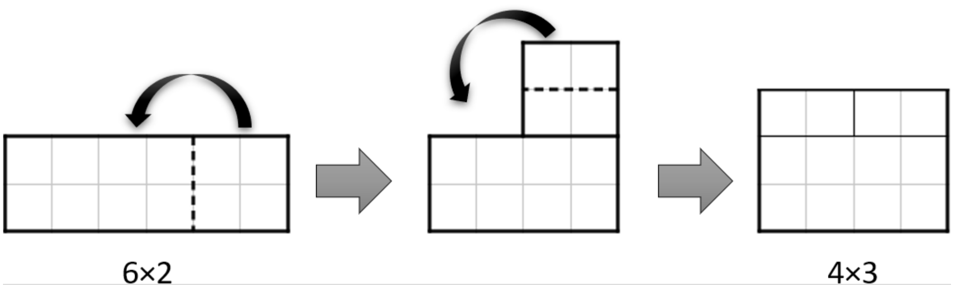

Figure 4
Combining and splitting rectangles.

From the mathematical point of view «we can recognize in this operations between rectangles the meaning of a relation of equivalence based on the fact that manipulating rectangles, the surface can remain the same or not» (Maffia & Mariotti, 2016, p. 9). Furthermore, in this relation of equivalence the expert can recognize the mathematical meanings of properties of multiplication. «For instance, we can recognize the commutative law of multiplication in rotations that invert the position of the sides of the rectangle» (Maffia & Mariotti, 2016, p. 9) (Figure 5).

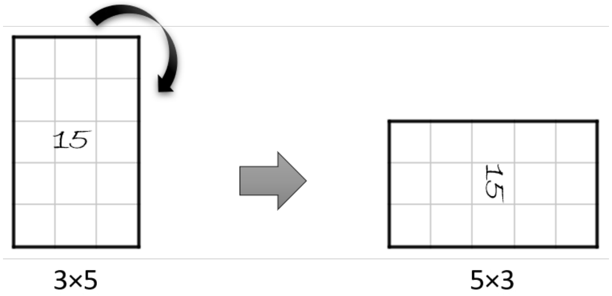

Figure 5
Example of rotation of a rectangle that could be associated to the commutative law.

Based on this kind of analysis of the semiotic potential, a sequence of didactical cycles has been designed and implemented. Students were asked to face different tasks with these artefacts (the table and the slip of papers). Collective discussions were orchestrated by their teacher to develop the relational meanings of multiplication properties, starting from unfolding the semiotic potential of the artefact in respect to the proposed tasks. In the following, excerpts from the discussions are used to exemplify the different teacher's actions.

Joint construction of shared-signs

Considering the objective of promoting the emergence of personal signs referring to the common experience with the artefact, two complementary actions can be defined by this objective: we call them back to the task action and *focalization* action. The first and crucial step in the semiotic mediation process consists in fostering the emergence of signs related to the actual use of the artefact, then a first class of situations can be characterized by the need of promoting the students' production of signs. In general, that need occurs at the very beginning of a classroom discussion, but it may also occur during the discussion flow: for instance, when students' contributions fail or stop to adequately feed the classroom discourse. In short, in all those moments in which the production of signs should start or re-start, the situation demand of an intentional and explicit intervention of the teacher aimed at:

– provoking students' production of personal signs related to the actual use of the artefact;
– constructing a shared context for these signs through evoking of the actual context of use of the artefact;
– obtaining contributions from all the students and in a number that is as large as possible.

This type of action is referred as *back to the task* action.

The objectives listed above are interrelated and the teacher's actions are often meant to pursue all of them simultaneously, even if from time to time one of them might be prominent.

Typical intervention can be «who want to report on the task that was proposed? ... How did you accomplish the task? … Which was the request of the task?»

The back to the task action is considered effective if it provokes a large number of contributions, however, not all the emerging elements might be related to the semiotic potential; that leads to the need of selecting the pertinent aspects of the shared meanings in respect to the development of the mathematical signs that constitute the final education goal. In those moments, when the selection of specific aspects is required, an intentional intervention of the teacher is required, aimed at

- highlighting specific (shared) signs produced up to that moment;
- selecting pertinent aspects of the meanings of these (shared) signs;
- circumscribing the reference of certain signs to specific aspects of the use of the artefact;
- supporting students' consciousness-raising of these key aspects.

This type of action is referred as *focalization* action.

In short, the objective is that of highlighting and limiting a part of the students' common experience with the artefact, in relation to its semiotic potential. In this case, gestures are often observed, sometimes specific aspects of the artefact's use are simulated.

In the following, short excerpts of a collective discussion illustrates the interlacement of interventions belonging to the two previous typologies. The excerpt below is taken from a discussion that followed an activity aimed at introducing the commutative law of multiplication.

Second graders were asked to choose a cell in Laisant table and to colour it. Then they had to prepare a slip of paper of the same shape and dimensions. Each pupil has his/her own copy of the Laisant table, paper and scissors for cutting the requested rectangle. After realizing the rectangular slip of paper, each child was asked to look for a cell in which the slip of paper could fit perfectly. Students moved their rectangles of paper on their copy of Laisant table and once found the right cell, they coloured it. Of course, there are two different cells fitting each rectangle: the one that was initially chosen by the student and the symmetric one that corresponds to the same multiplication with inverted order of factors. An exception is the case of squares appearing just one time in the table. The same procedure is repeated two times, each time starting from a different cell.

When all the children have finished their work, the teacher asks to share what they have noticed about the positions of the two coloured cells.

Excerpt 1

1. Teacher: Did you find out something? Who wants to say what he found? [Siro raises his hand] Ok, tell us.
2. Siro: The two twins are in the same position. Two are on the top and two are on the left. They are in the position because if you turn the paper then it becomes the opposite. This is mine. This is what is on my table.
3. Teacher: Ok! Let's start from this idea. Do you all agree that when someone turns the paper, then the cells go in the same position?
4. Many students: No.
5. Teacher: Give a look at your own table. Siro says that if you turn the paper, then the twins are in the same position.
6. Siro: Well. If I turn it this way [he turns his paper] it becomes... this one!

The teacher begins the discussion by asking what the children could "find out" (line 1). According to our characterization, such intervention can be classified as a *back to the task*. Actually, this is a very generic question, aimed at prompting students reports on their activities, and thus raising the production of personal signs that

are related to the activity performed with the aid of the artefact. This objective is suddenly realized thanks to Siro, who uses the word "twins" (line 2) to refer to a couple of coloured cells. This word has not a shared meaning and it is proposed in this moment, by this child, for the first time. He also describes the absolute positions of the cells in his table and he puts them in relation through the action of turning the paper (line 2). Maybe this is an action that he performed while solving the task.
After Siro's intervention, the teacher decides to draw pupils attention on what has been said and she *focuses* on the idea of turning the paper. Indeed, the teacher repeats part of the student's words. In particular, the word "turn"(line 3) and the the word "twins" (line 5) are mirrored. In both cases, it seems that the teacher recognizes the semiotic potential of these words with respect to the mathematical meaning of the commutative law that is the objective of this teaching intervention. Actually, the word "turn" may be related to the inversion of the factors in the multiplication, while the word "twins", referred to the cells, expresses the fact that the two colored cells have common features and correspond to each other by turning the paper.
However, since the other pupils (line 4) seem not convinced by Siro's intervention, in spite of its reformulation given by the teacher, the teacher asks the pupils to look back to their tables (line 5); in other words, students are asked to go back to what they did while solving the task. This prompts Siro to perform again the action of "turning the paper" showing it to his classmates (line 5).
This short excerpt shows how the teacher triggers the first step in constructing a shared context for the signs proposed by Siro (the words "turn" and "twins") through evoking the actual use of the artefact by the combination of the two actions: back to the task and focalization.

Towards the mathematical signs
As said above, back to the task and focalization are two complementary types of interventions, the former is meant to exploit the semantic richness of the context of the artefact, the latter is meant to focus the specific meanings that might be related to the expected mathematical meanings.
The mobilization, repeated and alternated, of these two types of action is expected to foster the construction of a shared web of signs, which on the one hand are anchored to the use of artefact, and on the other hand retain those key elements of their meanings which are pertinent in respect to the development of the mathematical signs that are the objective of the didactic intervention.
Nevertheless, the evolution towards the expected mathematical signs requires further interventions to get the required de-contextualization of the produced signs from the artefact and its use and the proper mathematical characterization. Two more types of intervention can be mobilized for attaining such evolution: *ask for a synthesis and offer of a synthesis*.
The unfolding of the semiotic potential – that is the emergence of shared and stable signs condensing the key aspects relating the artefact both to the experience of its use and to the Mathematics evoked by that use – requires the teacher intervention to promote the need of generalizing and de-contextualizing the meanings emerged. However, the process of generalization and de-contextualization of meanings cannot consist of simply replacing the produced signs (for instance the verb "turn" referred to the cells) with the appropriate mathematical signs ("inversion of factors"). New signs must be constructed and shared gaining a full mathematical meaning but at the same time keeping the specific key aspects related to their origin. For instance, the intrinsic dynamic of rotating the rectangular slip of paper is expected to

remain as a component of the mathematical meaning of commutative law, though its mathematical definition will not include explicitly the reference to rectangles. This evolution consists in a complex semiotic process requiring a direct intervention of the teacher aiming at:

- promoting the de-contextualization from the use of the artefact;
- promoting the generalization with respect to the specific tasks;
- maintaining in both the previous processes (de-contextualization and generalization) those aspects of the personal meanings recognized as pertinent to the target mathematical signs;

Such a complex semiotic process can be fostered by a teacher's intervention inviting students to summarize what has been discussed up to that point, and/or what students consider as shared in the ongoing collective discussion. We call this type of intervention ask for a synthesis. As a matter of fact, *asking for a synthesis* does not only induce students to make explicit their personal meanings, but it also induces them to condense different experiences in one sentence, and this may lead one to look for commonalities, and in so doing fostering generalisation. Moreover, syntheses produced during a collective discussion tend to involve signs previously emerged, but they also may include mathematical signs already in use or recently introduced by the teacher. In other words, sharing personal meanings through a synthesis forms the interpersonal space within which the teacher can introduce the point of view of mathematics, and eventually a standard terminology. The following excerpt shows examples of this actions.

Excerpt 2

55. Teacher: Marco came at the blackboard and he wrote the number 15 in this two twin cells [she points 3×5 and 5×3].
56. Siro: Yea! Because they are twins!
57. Teacher: So twin cells have always the same numbers?
58. *There are many students saying 'yes' but other students say 'no'.*
59. Teacher: Yes or no?
60. Many students: Yes!
61. Teacher: I will try with an example. Here I have 1, 2, 3 [she counts the squares along a side of the cell 3×4, it is colored on the table] in horizontal, and 1, 2, 3, 4 in vertical. What number do I put inside it?

The excerpt begins with the teacher focusing on what was done by one of the students, Marco (line 55), and relating it to the new sign "twins": he wrote the number 15 inside the two "twin cells" 3×5 and 5×3. This focusing prompts Siro to notice that this depends on the very nature of being twins (line 56). In line 57, the teacher poses a question that is no more referring just to the cells 3×5 and 5×3, but to all the couples of twin cells. Doing so, she is promoting a generalization with respect to the specific example. This can be recognized as a case of *ask for synthesis*. However this teacher's action does not sort the desired effect; indeed, not all the children agree on this fact (line 58). So the teacher invites the pupils to go back to the task (line 61). An effective ask for a synthesis may provoke the production of de-contextualized or general explanations, even if they are still not completely stated in mathematical terms.
When the discussion has set in motion the de-contextualization and generalization of meanings from the context of use of the artefact, but the evolution cannot yet be considered complete, the teacher may intervene with the *offer of a synthesis*, explicitly referring to the mathematical context and its meanings with the aim of:

- making explicit the relationships between mathematical meanings, and meanings constructed trough the classroom discussion;
- introducing the desired mathematical signs and providing a mathematical formulation;
- ratifying the acceptability and the mathematical status of a specific sign.

The excerpt below shows an example of teacher's offer of a synthesis.

Excerpt 3

43. Teacher: Ok, so: Nico says that it is not true for all the cells because the squares have no twins.
44. Siro: Indeed, I didn't say "all", I said just those.
45. Teacher: Those that are on that side.
46. Fabio: All the cells are a mirror. This one [he points the cell 5×5] reflects this and this, this and this [he points all the "twin" cells in the fifth row and column] while this one [he points 4×4] reflects this, this, this, this [he points the couple of cells in the fourth row and column].
47. Teacher: He is saying that all the squares are mirrors.

In this excerpt we can see that the process of generalization is going on through the discussion about the acceptability of the property of "having a twin" for all the cells. At the beginning, the teacher focuses on Nico's utterance (line 43) and this solicits an intervention by Fabio. According to him, all "cells" are mirrors (line 46), but then he just points the squares to say that they reflects the cells that are in the same row/column. So the teacher provides a synthesis (line 47): she re-interprets Fabio's words introducing the desired mathematical sign (square) and so ratifying the mathematical acceptability of Fabio's words. We can notice that this is not yet a pure mathematical statement because there is still a reference to the context of the artefact, nevertheless the process of de-contextualization has started, the appropriate mathematical signs have been introduced; now the process needs to go on.
This shows two interesting aspects: on the one hand, the offer of a synthesis is not meant to stop the semiotic mediation process, it may constitute an intermediate step in the evolution providing exemplar modes of de-contextualization and generalization; on the other hand, the complexity of the semiotic mediation process, involving the individual and the collective sphere, requires the teacher repeatedly alternating requests and offers of synthesis.

6 Conclusions

According to the TSM the didactic use of an artefact has a twofold nature: on the one hand it is directly used by the students as a mean to accomplish a task; on the other hand it is indirectly used by the teacher as a mean to achieve specific educational goals, for instance the construction of mathematical knowledge. Our contribution focused on the specific role played by the teacher in the management of the teaching and learning process, specifically we directed our attention to a particular phase of the general didactic arrangement – the phase of the collective discussion – outlining possible patterns in the modalities of interaction between the teacher and her/his students. After the analysis of a number of collective discussions, categories of possible actions were identified, leading to an explicit categorization of possible modes of teacher's intervention aimed at driving the semiotic process centred on the use of an artefact. Specifically, the two pairs of complementary actions introduced above, describe how, during a collective discussion, the teacher can foster both the joint construction of shared-signs and the evolution of these signs into the expected mathematical signs.

This model of the teacher's actions shades light on specific forms of mediation related to the teaching and learning process and on specific elements concerning the orchestration of the classroom discussion. It also clarifies what is expected by the teacher to let the artefact functions as a tool of semiotic mediation. The explicit characterization of the possible teacher's actions, makes possible to communicate and share them in the community of the teachers, contributing to foster teachers' professional development along this dimension, specifically focusing on the relevance of teacher's consciousness about her/his own role and specifically about the choices that s/he has to take during classroom conversations.

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